

a) Volume of slug = $A \cdot l$, mass $m = \rho \cdot \text{Volume} = \rho A l$

z -momentum = $m w_{\infty} = \rho A l w_{\infty}$

Wing pushes on air with vertical component L .

Time spent in slug $t = l/V$, z -Impulse of $L = L \cdot t = L l/V$

Using z -Impulse = z -Momentum:

$$L l/V = \rho A l w_{\infty} \rightarrow \boxed{L = \rho A V w_{\infty}}$$

b) KE in slug = $\frac{1}{2} m w_{\infty}^2 = \frac{1}{2} \rho A l w_{\infty}^2$

Work done by $D_i = D_i \cdot l$

Using work = KE

$$D_i l = \frac{1}{2} \rho A l w_{\infty}^2 \rightarrow \boxed{D_i = \frac{1}{2} \rho A w_{\infty}^2}$$

c) Alternative force model: $D_i = L \alpha_i = L \frac{w}{V}$

Substituting: $\frac{1}{2} \rho A w_{\infty}^2 = \rho A V w_{\infty} \cdot \frac{w}{V} \rightarrow \boxed{w = \frac{1}{2} w_{\infty}}$

d) $V' = 2V \therefore L' = \rho A V' w_{\infty}' = L = \rho A V w_{\infty}$

or $2V w_{\infty}' = V w_{\infty} \rightarrow \boxed{w_{\infty}' = \frac{1}{2} w_{\infty}}$

$D_i' = \frac{1}{2} \rho A w_{\infty}'^2 = \frac{1}{2} \rho A \left(\frac{1}{2} w_{\infty}\right)^2 = \frac{1}{2} \rho A w_{\infty}^2 \cdot \frac{1}{4}$

$\boxed{D_i' = \frac{1}{4} D_i}$, D_i scales as $1/V^2$

w_{∞} scales as $1/V$

Note: L' and D_i' here mean new 3D values, not the 2-D Lift/span. etc